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The Variation in the Calendar Time of the Equinox

By EDGAR W. WOOLARD

The Sun, in its annual apparent motion among the fixed stars, reaches at 6:38 P.M., Eastern Standard Time, on March 20, 1945, a point where it is commonly described as crossing the celestial equator from south to north. This event is now listed in the *Nautical Almanac*, among the "Phenomena," as "Sun enters Aries, Equinox"; previous to the 1940 issue, it was designated as "Sun enters Aries, spring commences," but we must remember that the seasons are reversed in the southern hemisphere so that there autumn is then beginning, and in response to frequent criticisms the entry was changed, even the usual adjective "vernal" being omitted in referring to the equinox.* Similar remarks apply to the other year points also.

The exact time of the equinox differs irregularly from year to year, and may occur on either March 20 or March 21 at Greenwich. Most of the variation is due to the discrepancy between the length of the calendar year and the mean period of the apparent tropical revolution of the Sun, which at intervals is nearly corrected by the insertion of an intercalary day in Leap Years; but a small part of the variation is due to effects of the precessional and nutational motions of the equinoctial point, and to irregularities and slow secular changes in the orbital motion of the Earth caused by the disturbing gravitational attractions of the other planets, which have the consequence that successive tropical years are not exactly equal in duration.

The equinox is technically considered to be the time at which the Sun, as it would be seen from the center of the Earth, is apparently at the same celestial longitude as the irregularly moving true equinoctial point of date—*i.e.*, the instant when the apparent geocentric longitude of the center of the Sun (affected by aberration) is 0°. Hence the exact time of the equinox may be found from the ephemeris of the Sun by interpolation, after precession and nutation have been added to the tabulated true longitudes in the *American Ephemeris* (which are referred to the mean equinox of the beginning of the year) and aberration subtracted, to obtain apparent longitudes. In 1945, *e.g.*, the apparent longitude of the Sun at 0^h GCT on March 20 is 359° 1′ 19″.9, therefore lacking 3520″.1 of having reached 360°, and on March 21 is 0° 0′ 55″.5, an increase of 3537″.6 in 24 hours, whence it becomes 0° at (3520.1/

*Cf. *The Sky*, 1940 March, p. 2.

3575.6) $\times 24^h = 23^h\ 38^m$ on March 20. The center of the disk of the Sun is just then entering the zodiacal sign of Aries; in general, however, it does not pass through the actual equinoctial point at all, because the equinoctial point is the intersection of the ecliptic with the celestial equator, and in technical astronomical usage the ecliptic is by definition the *mean* ecliptic—*i.e.*, the great circle in which the Sun would be instantaneously moving if only *secular* perturbations alone occurred in the orbital motion of the Earth. Because of periodic perturbations in the position of the instantaneous plane of the Earth's orbit in space, the Sun has day to day departures from the mean ecliptic so that its celestial latitude is not in general 0°, although it is never large; at the time of the equinox in March, 1945, the apparent latitude of the Sun is —0".46 (in 1944 it was —0".23, and in 1943 about +0".35). In reality, therefore, it is somewhat inaccurate to say that the equinox is the point where the Sun crosses the equator.

The time at which the equinox occurred in any given past year may be ascertained immediately, without calculation, by consulting the *American Ephemeris* or other almanac if one is obtainable for that year; but the time is easily calculated to within an hour or less, for any year from the remote past to far in the future, by means of several different tables that are now available: Zodiac tables have been constructed for determining the time of the Sun's entry into any zodiacal sign,[1] since this information is often needed in historical investigations; or, the longitude of the Sun may be quickly calculated with sufficient accuracy from special tables that have been prepared for the purpose,[2] or else it may be computed from the standard solar tables[3] but with periodic perturbations omitted. The results obtained from these different tables will not agree exactly, because they represent somewhat different approximations, but they will ordinarily differ by only a small fraction of an hour except for dates too far in the past or the future. In the accompanying table (Table I) are given the times of the March equinox for selected years during the 400-year period from 1700 to 2100; the times during the interval from 1895 to 1945 are taken from the *American Ephemeris*, those for earlier years are calculated from Neugebauer's zodiac table, and the ones for later years are computed from Newcomb's tables of the Sun, neglecting periodic perturbations and reductions to apparent longitude.

[1] See P. V. Neugebauer, *Astronomische Chronologie*, Bd. I, pp. 133-136, Berlin 1929. The particular tables constructed by Neugebauer appear in his *Hilfstafeln zur Berechnung von Himmels-Erscheinungen* (Taf. z. Astr. Chron. III), 2te Aufl., Leipzig 1925, and extend from 4000 B.C. to 2000 A.D.

[2] See P. V. Neugebauer, *Astron. Chron.*, pp. 50-53. The tables by Karl Schoch, *Planeten-Tafeln für Jedermann*, Berlin 1927, extend from 3400 B.C. to 2600 A.D.; the tables by P. V. Neugebauer, *Tafeln für Sonne, Planeten und Mond* (Taf. z. Astr. Chron. II), Leipzig 1914, cover the period 4000 B.C. to 3000 A.D.

[3] Simon Newcomb, Tables of the Motion of the Earth on its Axis and around the Sun, *Astron. Papers Amer. Ephem.*, vol. VI, Pt. I. These tables are for the interval from 1200 B.C. to 2400 A.D.

TABLE I

Year	—Time of March Equinox— GCT d h m	Day	EST		Interval since preceding equinox d h m
1700	20 14	20	9	a.m.	
1800	20 20	20	3	p.m.	
*1804	20 19	20	2	p.m.	
1895	20 21	20	4	p.m.	
*1896	20 2	19	9	p.m.	
1897	20 8	20	3	a.m.	
1898	20 14	20	9	a.m.	
1899	20 20	20	3	p.m.	
1900	21 1	20	8	p.m.	
1901	21 7	21	2	a.m.	
1902	21 13	21	8	a.m.	
1903	21 19	21	2	p.m.	
*1904	21 1	20	8	p.m.	
1905	21 7	21	2	a.m.	
1935	21 13 18	21	8:18	a.m.	
*1936	20 18 58	20	1:58	p.m.	365 5 40
1937	21 0 45	20	7:45	p.m.	47
1938	21 6 43	21	1:43	a.m.	58
1939	21 12 29	21	7:29	a.m.	46
*1940	20 18 24	20	1:24	p.m.	55
1941	21 0 21	20	7:21	p.m.	57
1942	21 6 11	21	1:11	a.m.	50
1943	21 12 3	21	7:03	a.m.	52
*1944	20 17 49	20	12:49	p.m.	46
1945	20 23 38	20	6:38	p.m.	49
*2096	19 13.8	19	9	a.m.	
2100	20 13.0	20	8	a.m.	

*Gregorian Leap Year.

The variations in the date of the equinox attract considerable popular interest. In ordinary years, the time of the equinox is approximately 6 hours later than in the preceding year, since the calendar year of 365 days begins about 6 hours before the Sun has completed a tropical revolution; but in Leap Years, the calendar year exceeds the tropical year by about 18 hours, and the equinox occurs 18 hours earlier than in the preceding year. However, the intercalary correction does not perfectly remove the accumulated discrepancy between the civil calendar and the Sun on any one occasion, and hence the variation of the equinox from year to year does not amount to exactly 6 or 18 hours. Irregularities are introduced into this variation by differences in the lengths of successive tropical years caused by nutation and by periodic perturbations in the motion of the Earth, although these effects are small and in the long run are not cumulative; the mean tropical year is also decreasing secularly at the rate of ½ second per century, but this is negligible. More important is the fact that the mean tropical year of $365^{d}\ 5^{h}\ 48^{m}\ 46^{s}$ exceeds an ordinary calendar year by $11^{m}\ 14^{s}$ less than 6 hours. The equinox in 1941, *e.g.*, was $5^{h}\ 57^{m}$ later than in 1940 because of the $5^{h}\ 49^{m}$ excess of the mean tropical year over the calendar year together

with 8^m due to the effect of nutation and perturbations. Finally, the perihelion of the Earth is continually moving eastward relative to the equinox, both because of the precession of the equinoctial point and because of a secular revolution of the line of apsides; consequently, successive tropical revolutions do not cover quite the same section of the elliptic orbit of the Earth, and therefore differ in duration by amounts which, though very slight, continue to be in the same sense for long periods of time and soon accumulate to an appreciable quantity.

The variations in the calendar time of the equinox are the resultant of (1) the variation in the actual lengths of individual tropical years, and their differences from the calendar years; and (2) differences in the lengths of individual calendar years introduced by the system of intercalation. The *average* calendar correction over the 400-year intercalary cycle is very nearly correct; but during the cycle itself an error is left after each intercalation, which averages about $44^m\ 56^s$ and continues to accumulate until the excess intercalation has been compensated by the omission of a bissextile day in 3 out of every 4 centennial years. As a consequence, the time of the equinox becomes on the average gradually earlier during the progress of the Gregorian cycle, occurring about 45 minutes earlier in each successive Leap Year until after a non-bissextile centennial year. Thus in 1896 the equinox came 17 hours earlier than in 1804. Then, because of the omission of an intercalary day in 1900, it occurred later each year until in 1903 the time was the latest during the whole 400-year period from 1700 to 2100; but during the long unbroken series of Leap Years from 1904 to 2096, the time gradually becomes about 36 hours earlier, giving the earliest equinox during the 400 years. A small residual error of nearly 3 hours still remains after the completion of a cycle (aside from effects of perturbations and nutation), and accumulates from cycle to cycle, since in 400 years the annual $11^m\ 14^s$ excess intercalation amounts to $74^h\ 53^m$ whereas only 3 intercalary days or 72 hours are omitted. The equinox in 2100 is therefore a little earlier than in 1700; but the actual amount depends upon the accumulated effect of the progressive increase in the longitude of perihelion. During a tropical revolution, the average rate of motion of the Earth relative to the moving equinox exceeds the average anomalistic motion by approximately $62''$ per year. At the end of a tropical year, the Earth is therefore not quite as far eastward around its orbit from perihelion as at the beginning; the exact interval of time that has been required for the tropical circuit depends upon the particular values of the true anomaly at the beginning and end of the period, and its difference from the mean tropical year is represented by the difference in the corresponding values of the equation of the center. For several thousand years at a time, the equation of the center, or departure from mean elliptic place, is progressively less at successive equinoxes, then progressively greater for a similar period, and therefore the accumulated difference from the mean does not in general

average out over a Gregorian cycle. At the equinox in 1700, the Sun was about 4′ farther ahead of its mean place than it will be at the equinox in 2100; hence the total length of the 400 tropical years comprised in this cycle is about 1.6 hours greater than 400 mean tropical years, and therefore only 1.3 hours less than the Gregorian calendar cycle instead of 2.9 hours. This explains the actual amount by which the equinox in 2100, neglecting nutation and short period perturbations, is earlier than in 1700; a more exact value of the difference than shown in the accompanying table is $1^h\,13^m$.

The date of the equinox depends not only upon the stage in the Gregorian cycle but also upon location on the Earth, since because of the International Date Line two dates are always simultaneously present on the globe (except just at the moment of Greenwich noon). The earliest time at which the equinox occurs anywhere on the Earth is in the 180th meridian time zone, east of the Date Line, where the time is 12 hours earlier than at Greenwich; and the latest is to the west of the Date Line, where it is 12 hours later than at Greenwich. During the period from 1700 to 2100, therefore, the earliest Civil Time on the globe at which the equinox occurs is March 19, at 2:00 A.M., in 2096, east of the Date Line; and the latest, March 22, at 7:00 A.M., in 1903, west of the Date Line. In the astronomical time reckoning used before 1925, these dates are, respectively, March $18^d\,14^h$ and March $21^d\,19^h$.

The September equinox, which ordinarily receives less attention than the March equinox, now occurs at varying hours on September 23 at Greenwich, but eventually will come to occur on September 22 in Leap Years as it is already doing in 75th meridian time. The interval from the March to the September equinox is approximately $186^d\,10^h$, but varies irregularly over a range of nearly half an hour because of perturbations in the motion of the Earth; the interval to the following March equinox is shorter, because the Earth is in the neighborhood of perihelion over this arc of the orbit and is moving more rapidly.

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A Simplified Method of Latitude Determination from Meridian Observations

By PAUL E. WYLIE

Many texts on spherical astronomy and on navigation recognize four cases in the solution for latitude of the observation at culmination of a heavenly body, *viz.:* (1) declination and latitude of opposite signs; upper transit; (2) declination and latitude of same sign, latitude the greater; upper transit; (3) declination and latitude of same sign, latitude the lesser; upper transit; (4) lower transit. Separate rules are stated for the determination of latitude under each of these conditions. These rules are not greatly honored by practical navigators, who, in preference to memorizing them, usually draw a projection on the meridian, approximately representing the conditions, and determine the latitude by reasoning based on the diagram.

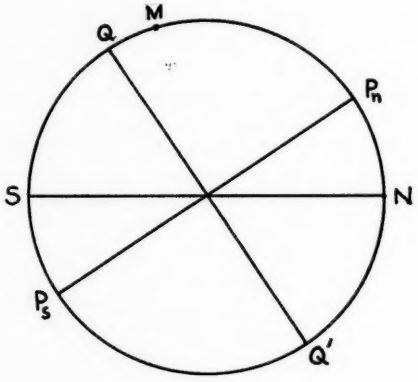

FIGURE 1

It is the purpose of this paper to show that the division of this problem into "cases" is unnecessary, as are also the reasoning and the drawing of diagrams. One simple formula will be developed, applicable, with simple qualifications, to *all* cases of meridian observation for latitude, including both those at upper and those at lower transit. Once the validity of this formula has been established, observers may thereafter apply it mechanically, just as navigators now apply rules in problems of compass conversion.

The accompanying figure (Fig. 1) is an orthographic projection of the celestial sphere upon the plane of the observer's meridian. The only condition is that the body observed shall be on that meridian. The figure,

as drawn, shows a body M, having a northern (+) declination of QM, a bearing of due south, and an observed corrected altitude (H_o), SQM. The latitude (ϕ), P_nN, is assumed to be a northern latitude. The horizon appears as SN, the polar axis as P_nP_s, and the celestial equator as QQ'. The diagram might just as well have been drawn for any other of the possible "cases." The inferences are the same, however the diagram is drawn, provided origins of measurement and signs of measurement remain unchanged from those we shall apply to the figure shown.

Let H_x be any arc of the meridian measured from the south point, S. The limit of measurement is 180°. Arcs above the horizon from S are to be considered positive.

Let D_x be any arc of the meridian measured from the intersection of the equator with the upper branch of the meridian, that is, from the point Q. The limit of measurement is 180°. Arcs measured north from Q are considered positive; south, negative.

Let ϕ represent the latitude of the observer; positive if north, negative if south.

Let δ represent the declination of the body M; positive if north, negative if south.

Then, from the figure, for the body M, since $QS = 90° - \phi$, $H_x = 90° - \phi + D_x$, from which we have

$$\phi = 90° - H_x + D_x. \tag{1}$$

Formula (1) is the general formula sought.

For the case figured, H_x = the observed altitude, H_o, and D_x = the declination, δ. The formula becomes

$$\phi = 90° - H_o + \delta. \tag{2}$$

Formula (2) gives the latitude in any case of upper transit, when the body bears south. When the body bears north, we must substitute $180° - H_o$ for H_x, in order to conform to our conventions of signs and measurement.

Formula (2), thus qualified, is adequate for the solution of every case of noon observation. This case is by far the most common application of the method.[1]

When a lower transit is observed, the method is the same, but, again preserving our conventions, we have $D_x = 180° - |\delta|$. D_x is given the sign of the declination.

A few examples will illustrate the method, which is perfectly general.

(A) Sun; observer in south latitude; $\delta = +20°$; Sun bearing north; $H_o = 60°$. By substitution, $\phi = 90° - 120° + 20°$; whence $\phi = -10°$.

(B) Sun; observer in southern hemisphere; $\delta = -20°$; Sun bearing north; $H_o = 75°$. $\phi = 90° - 105° - 20°$; whence $\phi = -35°$.

(C) Sun; observer in northern hemisphere; $\delta = -20°$; Sun bearing south; $H_o = 60°$. $\phi = 90° - 60° - 20°$; whence $\phi = +10°$.

(D) Star; observer in northern hemisphere; upper transit; $\delta = +80°$; star bearing north; $H_o = 30°$. $\phi = 90° - 150° + 80°$; whence $\phi = +20°$.

(E) Sun; northern summer solstice; observer near north geographic pole; upper

transit; Sun bearing south; $\delta = +23° 27'$; $H_o = 24° 27'$. $\phi = 90° - 24° 27' + 23° 27'$; whence $\phi = +89°$.

(F) Sun; northern winter solstice; observer near south geographic pole; lower transit; Sun bearing south; $\delta = -23° 27'$; $H_o = 22° 27'$. $\phi = 90° - 22° 27' - 156° 33'$; whence $\phi = -89°$.

(G) Polaris; lower transit; $\delta = +89°$; $H_o = 44°$. $\phi = 90° - 136° + 91°$; whence $\phi = +45°$.

The simplicity of the method will appear if the reader will check the foregoing examples with corresponding diagrams from which the latitude may be deduced. Experience indicates that, even at first application, errors are more likely to occur in reasoning from the diagram than in applying the formula. In practice, one need only memorize formula (2) and remember that, if the body bears north, $180° - H_o$ must be used in place of H_o, and that, if lower transit is observed, $180° - |\delta|$ must be used instead of δ, with the same sign as δ. No diagram need be drawn, nor need complicated special rules be memorized.[2]

NOTES

[1] This method, insofar as it applies to noon observations, has been presented by the writer in his "The Essentials of Modern Navigation," 5th Ed., 1943 (Harper), pp. 193-4.

[2] The writer wishes to acknowledge the interest and criticism of Dr. Frederick C. Leonard in the preparation of this paper.

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Personal Equation in Astronomy

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(Continued from page 76)

SECTION V

PERSONAL EQUATION IN OTHER BRANCHES OF ASTRONOMY

In the preceding sections personal equation has been discussed primarily in relation to meridian observations, since it was first noted and has probably been studied more thoroughly in these measurements than in any other field of astronomy. Although less publicized, the personal errors in other phases of astronomical work are quite as important. These errors appear in work with small field instruments and with equatorial telescopes, in measures of photographic data, and in other types of visual observations.

In the use of small field instruments, such as the portable transit or theodolite, the personal errors are of the same types as those found with larger instruments. Observations are confined mainly to the key- and chronograph, or the eye-and-ear method, and the forms of personal error found in these methods have been previously discussed. Measures with these portable instruments, however, are relatively less accurate than those made with larger telescopes and hence the personal errors

are of smaller consequence. E. Buchar[103] has given examples of the magnitude and type of personal equation found in theodolite observations.

Devices to obviate some of the personal errors in transit observations with small instruments were developed by S. P. Langley[55] and J. de G. Hunter.[98] Langley's first scheme, as projected in 1877, was to illuminate the telescope field with a glow lamp at regular intervals, the time of each flash being recorded on a chronograph. The rate of the flashes was set to accord roughly with diurnal motion and after the star entered the telescope field the observer made the final adjustment of rate so that the star appeared exactly bisected by successive stationary micrometer wires during the flashes. In 1903 he replaced the glow lamp by a shutter operating in front of the objective. With this arrangement the observer saw the star at timed intervals and attempted successive bisections by adjusting the operation of the shutter. It is probable, however, that the glow lamp idea would preclude observation of fainter stars, while the intermittent operation of the shutter mechanism might cause undesirable vibration of the telescope.

More recently (1932) Hunter adapted a shutter eyepiece to a portable transit for use in the Geodetic Survey of India. In 1936 the same idea was tried with a theodolite. In both of these instances the micrometer field is composed of many close lines etched on a glass plate. The shutter, operated by a chronometer, opens for 0^s.07 every three seconds, and the observer notes the position of the star in the field at these intervals. Hunter states that this method seems to eliminate part of the personal equation due to the motion of the star.

The Repsold micrometer has been adapted to some field instruments, the synchronous motor drive previously mentioned having been used by Watts on a Prin portable transit instrument in the world longitude operations of 1933.[100] The personal errors incident to this observing method have been discussed in Section IV.

Personal error also affects the work of the prismatic astrolabe. The errors arise in judging the instant when the two star images pass each other in the field, and the effect of star magnitude and rate of motion must be considered in connection with the reaction time taken to operate the punch key.

Machines to determine absolute personal equations with portable transit instruments are similar to those already described as in use with larger transit circles. Two methods for finding this value in connection with the astrolabe are given by Chandon, Volentat, and Gougenheim.[87] The first, used in conjunction with the astrolabe, consists of a collimator and a rotating prism which gives apparent motion to a fictitious star. The second, used without the astrolabe, is a machine which provides motion for two fictitious stars in a simulated field of view.

Dissatisfied with the traveling-wire and punch-key methods, however, A. Claude and L. Driencourt[63] suggest a return to eye-and-ear observa-

tions. Their scheme consists of a micrometer field of evenly spaced wires which can be rotated and also moved laterally by means of a rack. The object is to make the star coincide with a wire at the time of an audible clock beat. They state that this method simplifies the reduction of the observation, and that only one coincidence is necessary. M. A. Verschaffel[65] points out, however, that this method is subject to all the psycho-physiological errors found to be present in the eye-and-ear method.

Personal equation in micrometric measures with equatorial telescopes seems to be principally bisection error. In differential measurements, such as the separation of double stars, the bisection errors will cancel out, if the stars are of the same magnitude. If the stars are of unequal brightness, however, the bisection errors may not be the same and hence their residual will probably affect the measurd interval. In discussing this bisection error, R. M. Stewart[59] says, ". . . it will enter into all measures, differential or otherwise, where the objects measured are dissimilar, such as measurement of spectrum lines, or observations on stars of different magnitude." This residual bisection error in the measured distance between stars of unequal magnitude may be partially eliminated by observations both east and west of the meridian. For example, if the two stars have the same declination, the configuration east of the meridian should be almost the reverse of the configuration west, the degree of reversal being dependent upon the observer's latitude. Rotating the micrometer through 180° and remeasuring the distance will merely perpetuate this bisection error, but the use of a reversing prism should eliminate it.

Stewart[59] further states, "In the equatorial the personal error may vary according to the direction of the line joining the two stars to be measured." The results of a bisection estimate experiment, as given by R. S. Woodworth,[75] seem to substantiate this statement. The mean estimate of ten subjects as to the center of an interval was 0.509 in the horizontal coördinate and 0.486 in the vertical coördinate. Somewhere between the horizontal and the vertical, the mean bisection error changed from +0.009 to −0.014 and therefore it seems probable that this error varies with the rotation of the interval. Bisection error also seems to affect position-angle measures of double stars of unequal magnitude; the error apparently increasing in effect with the decreasing separation of the components. Salet and Bosler[57] have noted that the reversing prism obviates this error, and it seems probable that the reversing prism would negate all of the various forms of personal error mentioned above. A complete discussion of these errors in micrometrical measures is given by Aitken.[69] Unless a reversing prism is used in the observations, these personal errors will affect over a period of time the equatorial micrometer measures of satellites, double stars, and their derived orbits.

A purely physiological factor which further contributes to error is

present in observations of very faint stars where peripheral rather than foveal vision is used. The stimulus receptors in the retina of the eye are termed cone visual cells and rod visual cells.[114] The fovea is composed entirely of cone cells and these are connected singly to optic nerves. From the fovea to the periphery of the eye the number of cone cells decrease while the rod cells become more numerous. These visual cells lying outside the fovea are connected not singly but in groups to the optic nerves. Thus the foveal cells require a strong stimulus to initiate an optic nerve impulse, while the peripheral cells, working in groups, can initiate the nerve impulse with a weaker stimulus. In addition, the rod cells, in a dark-adapted eye, become about one thousand times more sensitive than the foveal cells. For these reasons it is possible to detect a faint object with indirect (peripheral) vision when it cannot be seen with direct (foveal) vision.

The group arrangement of peripheral cells is important with regard to acuity. If we denote the acuity (discrimination of form) of the fovea as ten units, then in terms of these same units the acuity of peripheral vision ranges from 2.5 at 10° from the center of the retina to about 0.3 at 40° from the center. In micrometric measures of faint objects involving peripheral vision, therefore, this lower acuity increases the uncertainty of the bisections. The uncertainties thus introduced are probably not systematic but merely tend to increase the accidental error of the measures.

Personal equation in occultation observations seems to have been noted first by Bessel. Experimentation since that time reveals that several factors influence the size of this personal equation. A dark-limb occultation observation follows a direct stimulus-response pattern and may be denoted as a simple reaction with some degree of preparatory set. In observing the disappearance of a star at the dark limb of the moon, it seems unlikely that the observer can achieve a very high degree of preparatory set since the instant of occultation is uncertain. With the dark limb visible due to earth-shine, however, the observer can see the limb approach the star and it seems probable that the degree of preparatory set attained should be greater. Attentional predisposition (see Section III) may also influence these reaction times. In the first instance mentioned above it may be supposed that the observer's attention is concentrated mainly on the stimulus, rather than on the response; *i.e.*, the preparatory set is of the "sensorial" type. With the dark limb visible, however, the observer can estimate when the occultation will occur, and thus probably can concentrate more upon response; *i.e.*, "muscular" preparatory set. From these premises then, it might be reasoned that in the same observer, the reaction time for a disappearance at the invisible dark limb should be greater than for a disappearance at the visible dark limb.

An experiment by E. C. Phillips[77] seems to substantiate this difference in personal equation. Using a machine to simulate occultations,

the mean reaction times determined from 210 observations by two observers were as follows: for disappearance at dark limb, 0ˢ.29 and 0ˢ.32; for disappearance with dark limb visible, 0ˢ.25 and 0ˢ.29. The average deviation of individual observations from the mean values was 0ˢ.03. While the difference is of the same order as the mean deviation, a similar change in the personal equation seems to be indicated by both observers. If this difference in reaction time when the dark limb is visible exists, then it will probably appear in occultation observations and in lunar positions based on these observations as a periodic error with a period of one synodic month.

The effect of star magnitude on personal equation in occultation observations has been investigated by Renz[28] and Przybyllok.[58] Both seem to indicate an increase in reaction time with decreasing brightness of the star. Probably Gill's explanation of magnitude equation in transit observations with key-and-chronograph could apply here (see Section II). E. Schembor[90] claims the existence of a color equation in occultation observations and gives one example of the correlation between an observer's color index and reaction times. More observational and experimental material is needed, however, to verify the influence of these factors on personal equation.

In the visual estimation of star magnitudes several psychological and physiological factors introduce a personal equation. The most important seem to be scale error, differential extinction of color, and the Purkinje effect. The Purkinje effect may be described briefly as follows: In daylight the brightest point in the spectrum appears to be in the yellow region, about λ 600, while in faint light the point of greatest apparent brightness shifts well toward green, about λ 540.[93] The explanation seems to be that in a dark-adapted eye the rod cells are most sensitive to light of short wave-length. Thus when the light intensity is reduced to the point where the cone cells cease to be the principal receptors, and the rod cells become effective, the Purkinje phenomenon occurs.

Minneart and van der Bilt[86] have noted an instance of personal equation which seems directly attributable to the Purkinje effect. In estimating the brightness of red stars over a range of roughly eight magnitudes, two observers had a difference which varied with magnitude. The estimates of both observers agreed at the extremes of the magnitude scale, but differed in between by an amount that reached a maximum of approximately 0.7 magnitude near the center. Minneart and van der Bilt suggest that the difference occurred because, as the star magnitudes increase, one observer changed from rod cell vision to cone cell vision earlier than the other observer. That is, the observers differed in the threshold levels of their cone cells. Due to the Purkinje effect, the first observer to change to cone cell vision would give a brighter magnitude estimate on a red star than would the observer still utilizing rod cell vision. On brighter stars, when presumably both observers had shifted to cone cell vision, their magnitude estimates were in accordance.

In making visual estimates of the brightest of Nova Puppis 1942 and several comparison stars, S. Gaposchkin[110] noted two forms of personal error: (1) His scale estimates were not linear functions of magnitude, and in extrapolated magnitudes this would introduce an error. (2) He saw the blue comparison stars relatively bright, and the red comparison stars relatively faint. Since the nova changed spectral type, being blue at maximum brightness, and thereafter becoming reddish, he notes that this personal error may have caused him to overestimate the nova's maximum magnitude and likewise to underestimate it as the nova became fainter. The comparison stars used varied over a range of 4.5 magnitudes, the bright ones being of spectral types O and G (blue), while the fainter stars were mainly K and M type (red). Therefore the second personal error mentioned cannot be attributed to the Purkinje effect, since the observed error is opposite to that which this phenomenon would introduce. Also it seems doubtful that the Purkinje effect would be noticeable in so limited a range of brightness. This overestimation of the blue stars, however, might possibly be explained on the assumption that the observer has better than average dark-adaptation of the rod cells.

In making these observations, Gaposchkin adopted the criterion that the nova and comparison stars be visible in the field at the same time, thus allowing complete intercomparison. In deriving the color equation then, he made the assumption that the stars were all reddened the same amount by differential extinction. This seems justified, since in view of the limited range of magnitude involved it is not probable that a change in the color order of differential extinction would occur. If stars with a greater range of brightness were being intercompared in the same field of view, however, the changing color order of differential extinction might introduce an error.

Another instance of personal error in magnitude estimation has been noted by E. C. Pickering.[26] The error appeared in the Harvard program to re-estimate the magnitudes of stars in the Bonn Durchmusterung, zone $+50°$ to $+55°$. The observational procedure was as follows: while the observer was making his magnitude estimate an assistant nearby would write down the Durchmusterung magnitude and then the estimate communicated by the observer. When the differences Durchmusterung minus Harvard were taken in the magnitude range 8.3 — 9.2, it was found that the distribution did not follow the normal curve, there being too many zero residuals.

Pickering suggested facetiously that the Harvard estimates had been influenced by the Durchmusterung magnitudes through thought transference from assistant to observer. It was found, however, that a scale error in tenths of a magnitude existed in the Durchmusterung, and that when the D — H residuals had been corrected for frequency distribution they conformed to the probability curve with only slight accidental dispersion.

The introduction of photographic methods in astronomy seemed to offer a type of observation that would be free from error. The discovery of certain physical limitations such as the effect of emulsion granulation on resolving power, and differential shrinkage of emulsion, however, have modified this early opinion. Schilt[104] has established that astrometric accuracy diminishes as the size of the configuration of reference stars increases and Land[109] asserts that this is due to the increased effect of relative film shifts with the size of the configuration. The errors which arise due to distortions of plate emulsion have been reduced by "normalized" plates; that is, plates in which the stresses in the undeveloped gelatine have been relieved before exposure. Again, in photographic work, the color of a star is known to affect slightly its position on the plate due to differential atmospheric refraction. This type of error has been discussed by Burns.[108] Schlesinger and Barney[102] have shown a change in the second Gaussian point of the objective with wave-length. This resulted in a change in the plate scale, which was progressively greater for red stars than for white stars. A discussion and analysis of some of the errors found in a photographic program has been given by V. Goedicke.[112]

It seems probable also that certain personal errors may arise in measuring photographic data. Quoting R. M. Stewart[59] again on differential bisection error, ". . . it will enter into all measures . . . where the objects are dissimilar, such as measurement of spectrum lines, or observations of stars of different magnitude." While automatic machines have probably cancelled the error in measuring spectrum lines, it seems possible that due to the varying density and diameter of star images this residual bisection error still enters visual measurements of photographic plates. Admittedly this error is small and may not be effective with the present precision of our measurements.

In observations with the Photographic Zenith Tube in 1929, F. B. Littell[83] noted a systematic difference of $+0''.076 \pm 0''.010$ between two observers. The differences between observers with this instrument are usually much smaller and about the size of the probable error. In this case Littell attributed the difference to the manner in which the reflecting basin of mercury was skimmed, and also to the fact that the mercury level was too low at times. In reducing the photographic data from this instrument, however, no systematic differences have been noted between the individuals measuring the plates.

The psychological errors effecting meteor observations have been set forth by C. C. Wylie.[105] First is the impression that a bright, distant meteor is very close. This may be attributed to the observer's association of brightness with nearness. Next are a fictitious lengthening and a fictitious increase in altitude of the meteor path, which seem due to the common over-estimation of acute angles; an established psychological phenomenon. The apparent sound of the meteor as the light is seen, and the apparent sight of some object accompanying the echo as

it rolls back along the path, seem to be other illusions due to association.

The foregoing discussion of personal equation illustrates some of the psychological and physiological limitations of man with which the scientist must always contend in his effort to increase the precision of fundamental measurements. In the field of astronomy, a multitude of ideas have been advanced in the attempt to obviate personal errors. As an observer on a meridan transit circle, the writer has been intrigued by one scheme especially, although it has numerous faults. Other observers who have endured the rigors of the sasons in an unheated building while studying the wonders of the heavens, may also find it the verbal expression of thoughts they have had on a cold winter night. As projected in 1894 by H. H. Turner,[38] the plan consists of a movable plane mirror mounted in front of a fixed telescope, the observer being under cover in a warm room and in comparative comfort, which is a condition of great importance in observing. With this arrangement the observer should be able to make all the necessary observations without moving from his seat at the telescope. While this ideal may never be attained, positional astronomers are continuing their efforts to remove those conditions which might affect an observer's personal equation. Since their existence was first noted nearly 150 years ago, personal errors have posed a definite problem in the exact science of astronomy, and their continued presence must be suspected until the time when, paradoxically, the influence of the observer is entirely eliminated from the observations.

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U.S. NAVAL OBSERVATORY, WASHINGTON, D. C., 1944 JANUARY.

The Amplement and the Coamplement of an Angle

By FREDERICK C. LEONARD

If the sum of two angles equals 90°, one is said to be the complement of the other; if their sum equals 180°, one is called the supplement of the other; and, if their sum equals 360°, one is referred to as being the explement of the other. Not knowing any established term for either of two angles, the sum of which equals 270°, I propose that the word *amplement* be adopted for the purpose and further that the complement of the amplement of an angle A, which equals $(A \pm 180°)$,* be called the *coamplement* of A. The following illustrations of the use of these expressions are taken from astronomy and trigonometry:

1. The altitude (h) and the zenith distance (z) of a celestial object are the complements of each other; so are the declination (δ) and the north polar distance (p).

2. If the azimuth (A) of a body is measured positively westward (*i.e.*, clockwise) from the south point on the horizon, and if the amplitude (M) is measured positively northward (*i.e.*, counterclockwise) from the east point, the azimuth and the amplitude are the amplements of each other.

3. At 0^h, local mean solar (or civil) time ($LM\odot T$), the sidereal time (θ) and the right ascension (α) of the mean Sun are the coamplements of each other, while, at any instant, the local apparent solar time ($LA\odot T$) and the hour angle (t) of the apparent Sun, and the local mean solar time ($LM\odot T$) and the hour angle (t) of the mean Sun, are respectively the coamplements of each other.

4. The right ascension (α) and the unhappily so-called "sidereal hour angle" (*"SHA"*) of an object are the explements of each other. (The term "sidereal hour angle," which apparently was first employed in *The American Air Almanac* for 1941, is inappropriate, (1) because the coördinate in question is not an hour angle and (2) because the concept is applicable not only to stars but to all kinds of celestial objects; *cf.* the discussion of the matter in Paul E. Wylie's "The Essentials of Modern Navigation," 5th Rev. Ed., 262-3, 1943. Mr. Wylie has recently

*Evidently $A + 180° = A - 180°$, since $360° = -360° = 0°$, insofar as the values of the trigonometric functions of an angle are concerned.

proposed in conversation a really pertinent term, *right descension* (symbol, b), to supersede the expression "sidereal hour angle." Benjamin Dutton, in his "Navigation and Nautical Astronomy," 7th Ed., 180, 1942, explains the origin of the term right ascension in this way: "The term right ascension is derived from the right sphere. For an observer on the equator, the sidereal interval between the rising of the vernal equinox and the rising of a fixed star is equal to the right ascension of the star." "Right descension," meaning literally, for the aspect of the *right* sphere, the sidereal interval between the *setting* of a star and that of the vernal equinox, is obviously, then, a far more apposite expression for the explement of the right ascension than is "sidereal hour angle.")

5. The absolute values of the trigonometric functions of any angle are equal respectively to the absolute values of the *same* trigonometric functions of its supplement, coamplement, and explement. Denoting the given angle by A and its supplement, coamplement, and explement by S, C, and E, respectively, we readily prove that

$$\sin A = \sin S = -\sin C = -\sin E;$$
$$\cos A = -\cos S = -\cos C = \cos E;$$
$$\tan A = -\tan S = \tan C = -\tan E; \text{ etc.}$$

These simple, general, and easily remembered relations are useful for evaluating the trigonometric functions of an angle, positive or negative, in any quadrant.

Department of Astronomy, University of California, Los Angeles,
1945 January 30

Chang-Hen, A Chinese Contemporary of Ptolemy

By Y. C. CHANG

This is, indeed, a very much belated biography! However, since the events of recent years have made the people of the Occident more keenly aware of the people of the Orient, it is natural and desirable that the early scientific thought of the Orient should become known to the Occident. This paper constitutes a step in that direction. Editor.

Chang-Hen (78-139 A.D.) was an astronomer of the Han Dynasty. To a Chinese, he does not seem to belong to the remote antiquity in the sense that we regard Ptolemy as an ancient astronomer. Nowadays, very few, even among astronomers, read the Almagest. But Chang-Hen's writings are studied by those of our students who claim to have any familiarity with the Chinese classical literature. In fact, Chang-Hen was talented in more than one way. Besides being an astronomer, he possessed literary ability of the highest order. The large amount of his writings that have come down to us through the long interval of

time bears eloquent evidence to this fact. Chang-Hen was fortunate in this matter, as the thorough destruction and burning of books ordered by the Chin Emperor had taken place about three hundred years before his birth.

A book on painting, written at about the eighth century, listed Chang-Hen as one of the six great artists of the Han Dynasty. It remarked that a relief map drawn by Chang-Hen was still to be seen at the time when the book was written. We find an interesting incident related there in connection with the artist-astronomer. The story goes that there was a most terrible looking animal, having the body of a hog with human head. The animal lived in the lake, but occasionally came out to bask on a rock by the shore. But as soon as Chang-Hen set up his easel and started to sketch, the monster immediately dived down into the lake, as if unwilling to have his ugly feature portrayed. Chang-Hen finally had to leave his pencil and paper aside and surreptitiously made a sketch of the animal on the sand with his toe. There may not be much truth in it. But the story goes to corroborate the fact that Chang-Hen was a great artist.

Like Ptolemy, Chang-Hen propounded the theory of the rotundity of the earth. In his book by the name of Hwen-Tien-I, the structure of the heavens is compared to an egg. He said that the earth is just like the yellow of an egg. But the metaphor was carried on a little too far. He went on to provide some kind of hypothetical fluid by which the earth is held in suspension, having probably the white of the egg in his mind.

Quite apart from the mechanism by which the earth is supported, this conception of sphericity leads to correct interpretations of celestial phenomena. By the way, the ancient Chinese divided a circle into 365¼ degrees, equal to the number of days in a year. So you will not be surprised to read the assertion that "182⅝ degrees of any great circle in the heaven are above us, and the other 182⅝ degrees below. Only one-half of the twenty-eight mansions (comparable to the twelve signs of the zodiac) are visible at any moment. The north celestial pole is 36 degrees above the horizon. So there must be a region around the north pole with a diameter of 72 degrees, that is always visible (the circle of perpetual apparition). Around the south pole, there is a similar region being always invisible. The distance between these two poles should be 182⅝ degrees."

As the gnomon has been used for astronomical purposes in China since time immemorial, it is rather to be expected that Chang-Hen should be able to explain correctly about the length of shadow and position of the sun at solstices. "The points of intersection of ecliptic and equator are the places where the sun is to be found at equinoxes. On such days, the length of the shadow cast by a gnomon is half way between its lengths on solstices." Of course this last expression should not be interpreted too exactly.

The projection used in the old Chinese celestial atlas is quite unique. The north pole is at the center of the atlas. All hour circles are straight lines radiating from the north pole. The parallels of declination are represented here as concentric circles. Consequently the parallels of southern declination thus projected become much larger than the equator. With such an atlas in mind, we shall understand what Chang-Hen meant when he said, "Half of the ecliptic is inside the equator, and the other half outside. The greatest separation between the ecliptic and the equator is 24 degrees. The daytime is shortest when the sun comes to the extremely south point of the ecliptic, being 24 degrees beyond the equator. The daytime is longest when the sun passes the northernmost point of the ecliptic, 24 degrees inside the equator." Chang-Hen's value as given here has been used in the discussion of the variation of the obliquity of the ecliptic.

If Hwen-Tien-I may be classified as a sort of spherical astronomy, then another book of Chang-Hen, entitled Lin-Hsien, is more descriptive in nature. Following Hwei-Nan-Tze, he believed that the universe is infinite in both space and time. In his stellar groupings, stars near the north pole are said to belong to the Imperial Palace. The twenty-eight lunar mansions in the surrounding regions are divided into four Palaces, designated as Blue Dragon (Virgo, Scorpio, Libra, Sagittarius), Red Bird (Sagittarius, Capricornus, Aquarius, Pegasus), White Tiger (Andromeda, Pisces, Aries, Taurus, Orion, Gemini, Cancer), and Black Tortoise, respectively. According to his enumeration, "there are 124 stellar configurations within the circle of perpetual apparition. Altogether, stellar groupings with names assigned amount to 320, containing 2500 stars. This does not include stars on southern sky which have been reported by navigators in remote regions. If we count the fainter stars, the number may be increased to 11,520." From the last mentioned figure, we may infer that the author must have had very good eye-sight, considerably above the average.

"The sun is like fire, and the moon is like water. Light radiates from fire, but water may cast shadows. The moon is formed on the side which is under the sunshine. The old moon is that part which is in the shade. When in opposition to the sun, the moon may fail to receive any sunshine. The light is then blocked by the earth." It seems evident that this artist-astronomer had well grasped the significance of lunar eclipse and phases.

The versatility of Chang-Hen's talent is further shown in the construction of little machinery and instruments. It is on record that he made a brass celestial globe, which was installed in a closed cellar and operated by running water. A person is to watch the celestial globe in the cellar; and when the globe shows that certain star just rises, or culminates, or sets, he is supposed to call out the phenomenon he sees to the observing astronomer outside. There was perfect agreement between the events occurring in the model and those in the celestial sphere.

The seismograph he constructed seems to be even more striking than his miniature planetarium. According to the most authentic history, this instrument assumed the shape of a vase, made of refined copper, with a diameter of eight feet. As it comes from the hand of an artist, one expects to find there plenty of engravings and embellishments with figures of animals and birds. Eight dragon-heads protrude radially from the side of the vase, each holding loosely a brass ball on its tongue. There is a frog with open mouth under every dragon's head. The complicated machinery is hidden in the body of the vase. Whenever there is an earth-quake, the mechanism would be disturbed and the ball would drop from the dragon's head into the mouth of the frog. The clanking noise of the metallic parts draws the attention of the person on watch. Usually only one dragon gives up its ball. From that situation, the direction of the center of the quake may be roughly located. Once a ball dropped from Chang-Hen's seismograph while no one in the city had noticed anything unusual. People began to ridicule this absurd contraption. To Chang-Hen's great satisfaction, a postman arrived a few days later and reported an earthquake in southern Kansu Province.

If Chang-Hen was able to make a planetarium which duplicated the motion of the heavens, and a seismograph which detected a slight earthquake at great distance, no wonder several other inventions have been attributed to his mechanical skill. But from our modern point of view, we may well question the truth of the record. For instance, Chang-Hen himself claimed that he made a wooden bird which could fly. A later writer, commenting on this matter, said that this wooden bird had feathered wings attached to the sides and mechanism in the body, and that it could fly a distance of several miles. Perhaps, besides the gunpowder and printing press, the art of mechanical flight was also invented in China. But I am rather inclined to think that this artist-astronomer might well have shared the flying dreams of Leonardo da Vinci fourteen centuries later. The happy result of successful flight for several miles could not be accepted without reserve.

As a true astronomer, Chang-Hen put up a staunch fight against astrologers of his time. He pointed out the numerous contradictions in the standard astrological writings and petitioned the emperor to ban all such books from circulation. However, the superstition of the time had its influence upon even the most advanced thinker. In Lin-Hsien, while expounding his theory of the moon, he related the story of the beautiful lady who stole and drank the elixir of her husband and escaped to the moon. Before the day of science has dawned, this may be just as good an explanation to account for the dark markings on the lunar surface. In proposing a reformation in the procedure of admission to civil service, Chang-Hen enumerated the occurrence of comets and earthquakes as evidence against the system that had been practiced in the past years. In the sixteenth century, Kepler still discussed the

connection between Halley's Comet and the Thirty Years' War. Similar unscientific remarks made in the first century need not cause much surprise.

Chang-Hen's literary works are mostly in the form of essays. His most well-known pieces described the three capital cities of Han Dynasty. For this work, it has been said that he spent ten years in collecting materials and in other preparations. One essay is a supposed dialogue with a skull which he met on the countryside. He made the proposal to restore the skull to life. The spirit of the skull, who professed himself to be Chuan-Tse, refused the offer, however. The spirit reasoned: "Life is busy, while death is a good rest. Now nothing could do me any harm. I have no craving for the wealth or honor of the world." Then silence ensued and Chang-Hen ordered his servant to have the skull buried.

Nevertheless, Chang-Hen himself was not as indifferent to social position as this essay would lead one to think. He was on intimate terms with the emperor and was once requested to air his views about the government. The chamberlains of the court were then most notorious. Lest Chang-Hen might say something against them, they maneuvered to have him appointed a magistrate in a distant province. Although he was already nearly sixty years of age, he managed to arrest the worst gangsters in the district and put all of them in jail. He died at the age of sixty-three. After the lapse of nearly nineteen centuries, the site of his tomb, his study, and his home, however, may still be identified in the province of Honan.

INSTITUTE OF ASTRONOMY, ACADEMIA SINICA, KUNMING, CHINA, DECMBER, 1944.

Hints to Star-Gazers

By JOHN G. KELLAR

In view of the efforts of eminent astronomers in France, England, and America to popularize astronomy, it is strange how far from the thoughts of the most of us are other worlds than our own. We leave to astronomers not only the labor of research, which is proper, but too much of the enjoyment of the fruits of that labor.

Many of our colleges and universities, it is true, have observatories, and their graduates majoring in astronomy are doing important research work. Besides, many observers affiliated with observatories and astronomical societies are pursuing special lines of research relating to comets, meteors and meteorites, occultations, variable stars, colors and spectra of stars, daylight photography of stars, novae, and auroral phenomena. Many amateurs even make their own telescopes. But somehow these enthusiasms—and how fine they are!—fail to trickle down to you and me, the general public. Perhaps it is because in our country

astronomy is not generally taught in the elementary schools. At ten years it is not too young to begin to learn about astronomy. Unusually fortunate are children in cities that have a planetarium, for they have the opportunity to begin to sense the rhythm of the universe, to absorb the meaning of the glorious spectacle of the heavens.

With rare temerity the writer, who is only self-taught in certain astronomical matters incidental to his profession, ventures to set down what seems to him the minimum that everyone should know about the universe, and how one should go about star-gazing for enjoyment. Necessarily he must recite many well-known facts, not expecting, however, that anyone will go to the length of remembering figures, but only their general import.

1. *The illusion of relative motion.* The earth rotates on its axis toward the east at the rate of a degree in four minutes. Being unconscious of this, we attribute to the stars an equal westward motion. Most of us have experienced a similar illusion. We have boarded a train heading east, let us say, beside another also stationary on a parallel track. While waiting for our train to leave we have become absorbed in reading. Our train begins to move eastward, but so smoothly as not to arouse our sense of motion. Looking out the window we think we see the other train moving westward, and are not brought back to reality except by an effort of reason, or until we notice that its wheels are not turning.

The transition from illusion to reality is often accompanied by a momentary dizziness; as when a person standing on a roof or a high tower watches too intently the clouds drifting overhead; or as first voyagers at sea, having become accustomed to heaving horizons, are dizzy for a time after landing.

If one watches a star, through a mounted erecting telescope, as it approaches, crosses, and recedes from the central vertical crosshair, and if he concentrates on the thought that the star is fixed, he will be able to capture, momentarily, the sense of the eastward motion of his meridian plane and of himself. It is even possible to do this by lying on the ground, face upward and with the head held firmly in position, and looking southward while a star goes over or behind some tall distant thin object. Try it when you are in Washington, using the Washington monument for a mark. But it is easier with a telescope, because you know it is fixed, and the apparent motion is magnified by the eyepiece.

2. *The so-called fixed stars.* All the stars are fixed, in the sense that, due to their inconceivably great distances, they are practically fixed for a considerable length of time.

3. *The position of the observer.* Our sun, and indeed the whole orbit of the earth, is a mere pin-point in the immensity of the universe. Consequently, in observations on the fixed stars—at least those made by engineers and navigators—it is immaterial whether they are considered

as made on the surface of the earth or at the center of the earth. This does not hold for the moon and the planets. They are not stars. Relatively speaking, they are objects close at hand.

4. *A few star distances.* As an illustration of star distances, a few are here taken from the list in Dr. Lindsay's paper in POPULAR ASTRONOMY for May, 1936, in which he refers to Dr. Schlesinger's *Catalogue of Bright Stars,* 1931.

The nearest fixed star, Alpha Centauri, or Rigil Kentaurus, is about 4¼ light-years distant, the light-year being the distance light travels in a year at the rate of 186,000 miles per second. Figure for yourself the distance in miles. It is not worth remembering, but it is easy to remember that the distance to the sun is 93 times a million miles, while that to the nearest fixed star is 25,000,000 times a million miles.

The nearest star visible to those of us who live in Washington is Sirius. Its distance is 8.7 light-years, or around 50,000,000 times a million miles, and its luminosity is 27 times that of our sun.

Dr. Lindsay's table is constructed for star distances of from 32.6 to 3259 light-years. There are few stars nearer than the lower figure.

Polaris, of especial interest to engineers, is 271.6 light-years distant, and its luminosity is 724 times that of the sun. Rigel, the brightest star in Orion, is 543.2 light-years distant, and its luminosity is 17,700 times that of the sun.

5. *The constellations.* The constellations—notwithstanding the poets to the contrary—are not like tapestry hung in the sky. Such groupings are artificial. The stars in a constellation appear to preserve their relative positions solely because of their great distances from us—not only great, but greatly varying. For instance, Rigel is about twice as far from us as is Betelgeuse in the same constellation.

6. *The celestial sphere.* The figures given above should convince anyone that there is no such thing as a celestial sphere on the surface of which celestial bodies reside. Such a concept is a figment of the imagination, or rather it is an invention of mathematicians who have here to deal with angles but not with distances. Adopting this concept, which is convenient, everyone will probably agree that the center of such a sphere is the center of the earth. But what about the radius?

Should you ask the man in the street about the length of the radius, he might say, "I don't know;" or "It is any distance you like." If mathematically inclined, he might say, "It is unity." These answers amount to the same thing, but the second definition seems to be the best.

But if for the celestial bodies we should substitute their images, bringing the images inward of projecting them outward to some fixed distance along the lines of sight, could there not be a celestial sphere? It would seem so. That is what the engineer does when he sights the stars, for sidereal focus is constant for his telescope. In that case, then, the radius of the celestial sphere is the small distance from his eye to the

reticule bearing the crosshairs. Such a small celestial sphere! A celestial globe may just as well be considered a celestial sphere.

There is an interesting problem in navigation in which a definite length is assigned to this radius. It is the great-circle problem. Take for an example the great-circle or shortest distance from London to Panama City. First the zeniths of the places are substituted for the places. Then the arcs from zenith to pole are found by subtracting the latitudes from 90°. Then the angle at the pole included by these arcs is found as the difference of longitudes. The third arc, which is that connecting the two zeniths, completes an astronomical triangle, which when solved by trigonometry gives the arc from zenith to zenith as 76° 28′, or 4588′. This is on any celestial sphere. Finally, by choosing that celestial sphere whose radius is equal to the mean radius of the earth, on which 1′ of the arc of a great circle is equal to a mile, of the kind used at sea or in the air, the required distance is seen to be 4588 miles. Here again we have an actual celestial sphere.

7. *The altitude-azimuth system.* The prospective star-gazer has the altitude-azimuth system ingrained in his senses. He lives in it. The pull of gravity gives him his sense of the vertical, and of the plane of the horizon perpendicular to the plumb line. The varying muscular efforts of sighting high or low stars give him the sense of altitude, of which the primary unit is 90°, horizon to zenith.

The separation of the leaves of a book stood on end on a table illustrates azimuth planes perpendicular to the horizon. Open the covers 180° apart and leading away from you toward the north. Your position is at the bottom of the center line of the back. The farther cover is a plane of azimuth 0°, the nearer one is a plane of azimuth 180°. The azimuth of any point in the first plane, be it near or far from your position, high or low, above or below the table, provided that it does not pass the zenith or the nadir, is 0°. If any page stands 30° to the right of this reference plane, the azimuth of any point in it is 30°. East is 90°, south 180°, west 270°, and so on around to north again, 360° or 0°. The azimuth angles, which are angles between planes, may be seen, by looking down between the pages, as the diverging bottom edges of the pages. They are the angles that would be measured with a theodolite, in the plane of the horizon.

This system will be needed if you wish to tell some one in just what part of the heavens you have seen a particular star. But it is not the system in which positions of stars are furnished by the *Nautical Almanac*. Most of the problems in navigation are concerned with the transfer from one system to the other.

8. *The declination-right ascension system.* The star-gazer will have to start with the *Nautical Almanac,* which furnishes declinations and right ascensions. Declinations are reckoned, like latitudes, north or south of the plane of the equator. Right ascensions are reckoned east-

ward from a plane passing through the axis of the earth and the vernal equinox. The star-gazer may consider that the plane through the vernal equinox is stationary for the current year; also that right ascensions are fixed for the current year. He will find them in the table of mean places.

To visualize this system, draw, on a fairly large sheet of cardboard, two perpendicular lines intersecting at the center, which may be called C. Call one NS, the axis of the earth, N being the north pole and S the south pole. Call the other WE, W being the western and E the eastern point of an assumed equatorial diameter. With SN leading away from you and toward the north, W may also be the vernal equinox. The illustration will be given for latitude 20° N. Therefore the altitude of the north pole N will be 20°, and the plane of the equator, passing through WE, will lean 20° toward you. Cut half-way through the card under NE so that the card will fold easily.

Lay the farther edge of the card on a thick book so as to give it an upward slant away from you of about 20°, with N at the top. Make a large dot on the line NS at about one-third the distance from C toward N. This is your position in the universe.

Weight the left-hand half of the card. It represents the fixed plane through the axis of the earth and the vernal equinox. The right-hand half is the plane of the meridian. Contrary to the evidence of our senses, it is not fixed, but rotates about the axis of the earth in a clockwise direction, facing northward, at the rate of 360° per sidereal day, or 15° per hour. It carries with it the observer's true horizon, which is always perpendicular to his meridian plane. The true horizon passes through the center of the earth, or it may be considered to pass through the observer's position on the earth.

In making this rotation the meridian plane, once each sidereal day, sweeps through the vernal equinox, which defines right ascenion 0^h. It sweeps successively through all the stars, and for such as are listed in the *Nautical Almanac* the corresponding local sidereal times are those indicated by the listed right ascensions, in much the same manner as the hour hand of a sidereal clock sweeps over the twenty-four hour spaces of the dial. On what we may call the celestial clock, however, there are no figures 1, 2, 3, . . . 24 at regular spaces. Instead there are stars at irregular spaces.

But to the star-gazer, who cannot easily rid himself of the feeling that his meridian plane is fixed, or of the idea, corresponding to his sense perceptions, that all of the universe is rotating about the axis of the earth, the illusion seems easier to understand than the reality. Adopting his point of view, therefore, we have the following system stated in terms of apparent motions:

The meridian and horizon planes are fixed.

All of the planes intersecting in the axis of the earth, and passing through the vernal equinox and the stars, rotate about the axis of the

earth in a clockwise direction, facing south.

Instead of the meridian sweeping through the stars, the stars come successively to the meridian or, as we say, "transit" the meridian.

9. *Hour angles.* The hour angle of a star at any given local sidereal time is the interval of time elapsed since culmination. Since it culminated at the time indicated by its right ascension, the hour angle is equal to the local sidereal time minus the right ascension.

If a star was on the meridian one (sidereal) hour ago, its hour angle at the moment is 1^h or 15°. If it will be on the meridian one hour from now, its hour angle is 23^h or 345°.

10. *To find the approximate local sidereal time.* The proposed rule of thumb, for mental calculation, involves uncertainties as follows:

By not taking account of leap years and common years it is possible to be in error up to three minutes. By not taking account of the unequal lengths of the months, it is possible to be in error a few minutes. But the approximation is sufficiently accurate for finding bright stars without an instrument. An illustration will be given for June 15, any year, at 8:00 P.M. or 20^h civil time.

h	m	
6	40	Sid. T. at midnight between December 31 and January 1 of any year.
20	00	Mean time interval from midnight to hour of observation.
	3	(200^s, dropping 20^s) sidereal gain for 20^h.
2	43	Sid. T. of January 1, 20^h C.T. (24^h dropped).
10	00	Sid. gain for 5 months, January 1 to June 1.
	56	Sid. gain for 14 days, June 1 to June 15.
13	39	Sid. T. at 20^h C.T., or 8:00 P.M., of June 15.

The initial sidereal time to keep in mind for all such problems is $6^h\ 40^m$ for midnight on December 31.

The rule for the gain in sidereal time over mean solar time is 2^h per month, 4^m per day, and 10^s per hour.

The time of observation must be expressed in, or reduced to, local civil time. The correction from standard to civil time must be added when the observer is east of the standard meridian, and subtracted when he is west of it.

11. *To find the zenith distance of transit.* The first thing a star-gazer wants to know, when starting out to find one or more bright stars, is his local sidereal time, roughly, and he may either set his watch to show that time or he may note how fast or slow the watch is. The next thing he will need to know, for any particular star, is its declination, in order that he may compute mentally how many degrees north or south of the zenith it has transited or will transit. The starting point is the declination of his zenith, which is equal to his latitude. The rule is very simple: Go from the declination of the zenith to that of the equator, which is 0°, and then from the declination of the equator to that of the star. Combine the two results algebraically. Some examples will be

given for latitude 45° N., that being the latitude of the accompanying map, for stars having the following declinations:

Rigel, 8° 17′ S.	Aldebaran, 16° 24′ N.
Sirius, 16° 38′ S.	Spica, 10° 52′ S.
Dubhe, 62° 04′ N.	Beta Crucis, 59° 33′ S.

To the equator, 45° S; thence to Rigel 8° 17′ S; total 53° 17′ S.
To the equator, 45° S; thence to Aldebaran 16° 24′ N; total 28° 36′ S.
To the equator, 45° S; thence to Sirius 16° 38′ S; total 61° 38′ S.
To the equator, 45° S; thence to Spica 10° 52′ S; total 55° 52′ S.
To the equator, 45° S; thence to Dubhe 62° 04′ N; total 17° 04′ N.
To the equator, 45° S; thence to Beta Crucis 59° 33′ S; total 104° 33′ S.

Since the zenith distance of the horizon is 90°, Beta Crucis at culmination, the nearest point to the zenith, is 14° 33′ below the horizon. Therefore this star is never visible in latitude 45° N.

Dubhe, on the other hand, may be seen even at lower culmination.

To the pole, 45° N; thence to lower culmination, 90° — 62° 04′, or 27° 56′ N; total 45° + 27° 54′, or 72° 56′ N.

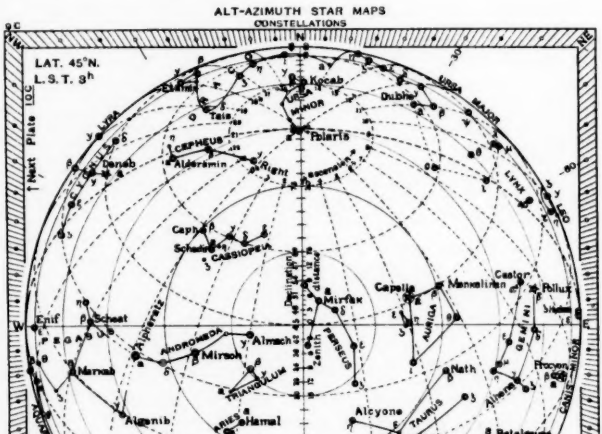

FIGURE 1

12. *Rising and setting of stars.* The reader will do well to find the meridian zenith distances of stars in which he is interested over a wide range of declinations for the latitude of his home town. He will find that some stars never rise, and so never set, that others rise and set, and that others are always visible. Those south of the equator, like

our sun in winter, rise south of east, if at all, and set south of west. Those north of the equator rise north of east and set north of west, or else they never set.

13. *The colatitude rule.* The colatitude of a place is 90° minus the latitude, in north latitudes. For instance the colatitude of Washington is 90° — 39°, or 51°. The rules for rising and setting of stars for an observer in Washington are as follows:

Stars of declination —51° or farther south never rise.

Stars of declination —51° to +51° rise and set.

Stars of declination +51° or farther north never set.

The Southern Cross, with declinations lying between —56° and —60°, is never seen in Washington, but may be seen from places in the United States south of latitude 30° N. All of the bright stars in Ursa Major, except the southernmost, Benetnash, declination 49° 36′ N, may be seen in Washington in all parts of their circuits about the pole. In Boston, latitude 42½° N, colatitude 47½° N, all of them are visible throughout their circuits.

14. *Suggested procedure for star-gazers.* Before going for an evening walk, compute mentally the local sidereal time, and either set your watch to it or note the watch correction. From the *Nautical Almanac* select a few bright stars having right ascensions within a few hours of this time, for it is best to begin with stars not too far from the meridian; and apply the colatitude rule to be sure that they will be above the horizon. For stars near lower culmination look in the list for right ascensions 12 hours later or earlier. Write on a card the right ascensions and declinations of stars that you hope to spot.

Arrived at the point of observation, in north latitude, let us say, plunge a cane in the ground so as to incline it downward toward the south by the amount of the latitude, thus pointing to the south pole; or, looking northward, pointing to Polaris, which is near enough to the pole for the purpose. This represents the axis of the earth. Picture the stars as moving in planes perpendicular to the cane, clockwise looking south, anti-clockwise looking north. For each star mentally fix the point in the heavens where it has culminated or will culminate, south or north of the zenith. Judge as best you can the rising and setting points, or, in the case of a star far north, the eastern and western points of its apparent elliptical path.

Having spotted one star in a constellation, use its right ascension and declination to help find another in that region. For instance if you have on your list

Regulus	R.A. 10:05	Dec. +12° 15′
Denebola	11:46	+14° 54′

and have located Regulus, look for Denebola at a point $1^h\,41^m$ later (eastward), and 2° 39′ farther north.

15. *Sea voyages.* The sea voyager from North to South America or

on any south-bound trip, has a fine opportunity, night after night, to watch the northern constellations descend toward the horizon, and the southern ones to ascend, at a rate corresponding to his own progress southward. How much each day? Well, if his ship makes a southing of 300 miles per day, the answer is 300 minutes, or 5 degrees, a noticeable amount. And on the return voyage, though he may regret bidding goodbye to the newly-made friends in the far south, there will be an added delight in greeting old friends in the far north.

May such enjoyment come to all of you.

BETHESDA, MARYLAND.

The Planets in April, 1945

NOTE: Greenwich Civil Time is employed unless otherwise stated. To obtain Eastern War Time subtract 4 hours, Central War Time, 5 hours, etc. The phenomena are described as they are to be seen from latitude 45° N. The data are taken chiefly from the *American Ephemeris and Nautical Almanac*.

Sun. The sun will advance northeastwardly quite rapidly during April, changing in declination from +4° 19′ to +14° 35′, and in right ascension from $0^h\ 40^m$ to $2^h\ 27^m$. Its path for this period extends from Pisces into Aries.

Moon. The phases of the moon will occur as follows:

	d	h
Last Quarter	April 5	19
New Moon	12	29
First Quarter	19	8
Full Moon	27	11

It will be nearest the earth for the month on April 12 and farthest from the earth on April 25.

Mercury. At the beginning of the month, Mercury will still be near a point of greatest elongation east and consequently will probably still be visible just after sunset for northern observers. Its motion becomes stationary on April 3 and retrograde after that. It will move rapidly toward the sun, reaching inferior conjunction on April 14, and be invisible in the sun's rays for the remainder of the month.

Venus. After having been the brilliant evening star for a few months, Venus will move westward during this month. This will cause it to approach the sun rapidly and bring it to a position of inferior conjunction on April 16. Venus will therefore be invisible or at least too close to the sun for good observation throughout the month.

Mars. Mars, moving eastward at nearly the same rate as the sun will maintain practically the same position relative to the sun so far as rising time is concerned. It will rise approximately two hours before the sun and a little south of the rising point of the sun. During the month it will move from about eleven degrees south of the equator nearly to the equator. Mars will be in the constellation Aquarius.

Jupiter. Jupiter will still have a slow retrograde motion during the month. It will rise a little before sunset and hence be above the horizon practically all

night. It will be near the equator and will be easily recognized because of its brilliancy, which is greater than that of any of the stars.

Saturn. Saturn will be a short distance west of the meridian at sunset. It will be about fifteen degrees west of Castor and Pollux and will not differ much from them in magnitude.

Uranus. As during March, Uranus will maintain a position in Taurus about two hours west of Saturn. It will, therefore, be observable in the evening hours, somewhat longer because of its high northern declination.

Neptune. Neptune, having passed opposition late in March, will rise before sunset during April. It will be in the constellation Virgo.

Pluto. The position of Pluto, as given by the Nautical Almanac office, for April 15 is $\alpha = 8^h\,48^m4$, $\delta = +24°\,05'$.

Occultation Predictions for April, 1945

(Taken from the *American Ephemeris*)

Date 1945	Star	Mag.	IMMERSION Greenwich C.T.	*a*	*b*	Angle E from N	EMERSION Greenwich C.T.	*a*	*b*	Angle E from N
Occultations Visible in Longitude +72° 30′, Latitude +42° 30′										
Apr. 2	73 B.Scor	6.4	9 10.1	−2.8	+1.0	52	9 57.9	−1.4	−3.0	346
3	68 B.Ophi	5.9	4 20.9	−1.1	+2.6	62	5 5.8	+0.2	−1.0	338
16	ζ Taur	3.0	15 10.1	0.0	+1.6	69	16 8.0	−0.4	+1.5	261
26	80 Virg	5.8	7 27.6	−1.3	−1.3	86	8 30.6	−0.6	−2.4	326
30	123 B.Scor	6.5	4 37.9	−1.0	−0.1	131	5 55.9	−2.0	+0.4	273
Occultations Visible in Longitude +91° 0′, Latitude +40° 0′										
Apr. 2	73 B.Scor	6.4	8 17.0	−2.3	+0.7	86	9 35.9	−1.6	−1.4	322
3	116 B.Ophi	6.3	12 20.8	−1.9	−2.0	135	13 26.8	−1.1	−0.4	241
16	ζ Taur	3.0	15 11.5	+0.5	+1.5	57	16 0.6	−0.1	+1.0	276
26	80 Virg	5.8	7 0.2	−1.9	−1.4	109	8 21.1	−1.1	−2.1	312
30	123 B.Scor	6.5	4 34.2	+0.7	−2.3	171	5 11.7	−2.6	+3.3	234
Occultations Visible in Longitude +120° 0′, Latitude +36° 0′										
Apr. 2	72 B.Scor	6.4	7 39.2	−0.3	−0.5	141	8 47.2	−1.8	+1.0	271
2	88 B.Scor	6.4	10 50.3	−3.5	+1.7	62	11 48.5	−1.2	−2.9	345
3	116 B.Ophi	6.3	11 37.9	−0.8	−2.8	166	12 25.3	−3.4	+2.0	228
17	14 B.Gemi	6.0	6 52.8	−0.8	+1.2	31	7 17.2	+1.2	−3.1	336
26	80 Virg	5.8	6 23.0	−0.6	−2.7	168	7 30.2	−3.1	+0.2	264
Occultations Visible in Longitude +98° 0′, Latitude +30° 0′*										
Apr. 2	73 B.Scor	6.4	8 01.3	−1.6	−0.4	122	9 32.4	−2.4	−0.6	292
2	88 B.Scor	6.4	11 57.8	−2.8	+1.7	47	12 45.1	−2.0	−4.2	344
3	116 B.Ophi	6.3	12 33.7	..	..	164	13 12.0	−1.5	+2.5	214
4	21 G.Sgtr	5.7	13 38.7	−1.7	+1.1	45	14 40.2	−2.3	−2.7	314
18	BD+22°1531	6.9	2 14.1	−1.4	−2.2	122	..	..	..	..	...
19	BD+21°1753	7.3	7 12.3	+0.7	−2.1	144	..	..	..	..	...
23	BD+ 6°2529	7.5	22 43.0	−0.3	−1.3	150	..	..	..	..	...
26	80 Virg	5.8	7 04.4	−1.7	−2.2	139	8 29.8	−1.8	−1.5	287
30	131 B.Scor	5.6	4 33.0	−1.5	+2.2	67	5 22.1	+0.1	−1.9	341

*Computation by George R. Schaefer and Paul Herget, communicated by Commodore J. F. Hellweg, Superintendent U.S. Naval Observatory.

The quantities in the columns *a* and *b* are given for the purpose of making these predictions useful for any place within 200 miles of the point indicated.

The procedure is as follows: Subtract the longitude of the point given from the longitude of the place in question; multiply the result in degrees, taking the signs into account, by the quantity under a for the star to be observed; similarly, with the latitude, using b; apply the sum of the products, with its proper sign, to the Greenwich C.T., and obtain the predicted Greenwich Civil Time for the phenomenon at the place of observation. To obtain Eastern Standard Time it is necessary to subtract five hours; Central Standard Time, six hours, etc.

METEORS AND METEORITES

Meteor Notes from the American Meteor Society

By CHARLES P. OLIVIER, President

This paper will consist of results on four fireballs, the first two of which turn out to be fragmentary and unsatisfactory, the last two fairly good solutions. Before starting, it would be well to say that we have mailed out Reprint No. 65 to active workers and to those who have paid their dues; others will receive it when the $1 dues for 1945 are paid. The annual report is being held up to give all time to get in their observations made in last or previous years. Will all members do so without further delay?

Bright Meteor of 1932 April 7/8

On this date at 12:41:47 E.S.T., a bright meteor or fireball was reported from two ships and by one observer in New York City. As the first two gave all the coordinates (at least approximately) and the last might serve as a check, attempts were made at intervals by two members of the staff and lastly myself to calculate the path. As it turned out, from the rather unfavorable positions of the ships with regard to the path, small errors in coordinates were more harmful than usual, so the time spent on the case was largely wasted. The one useful datum is the end point which, by two averaged approximations, I find as 90 ± 24 km over $\lambda = 74° 48'$ W, $\phi = 39° 38'$ N, the longitude being better determined than the latitude. The path was very roughly E to W, and the slope probably less than 20°. Its duration was 4 ± 1 sec., and it burst. Its path was quite long, and the observers differed as to its color. Two of the observations appeared in the *Hydrographic Bulletin*. They are not copied here as my results are too meagre and further work on the case useless.

Fireball(s) of 1936 May 21/22

The following two (abstracted) observations were received from Dr. Frederick Slocum at Van Vleck Observatory, Middletown, Conn., and Dr. Roy K. Marshall then at Chambersburg, Pa. The observations hence had the advantage of being made by two astronomers, but also the handicap of twilight skies. S1, $\lambda = 72° 40'$, $\phi = 41° 33'$, 8:03:40 P.M., E.S.T., $\alpha_1 = 52°.5$, $\delta_1 = +52°$, $\alpha_2 = 25°$, $\delta_2 = +43°$, magn. —5, duration 5 sec., yellow-orange, sparks, pear-shaped. S2, $\lambda = 77° 40'$, $\phi = 39° 56'$, 00:59:10 U.T. (of May 22), first seen but not true begining at $\alpha_1 = 277°.5$, $\delta_1 = +50°$, ended at $\alpha_2 = 282°.5$, $\delta_2 = +27°$, magn. —6 or brighter, duration 2 sec., color of Capella, no burst or train. Only Vega visible in twilight for comparison.

Despite the stated 4-minute difference in time, there is always a *chance* for a

meteor to be timed by an inaccurate watch, etc., and as other data seemed so similar, a solution was made by Miss E. F. Reilly. This gave results of no possible value or probability, so there must have been two fireballs. Yet their great similarity in physical appearance, etc., makes me believe that they belonged to the same radiant, which as a consequence I have worked out. Its coordinates are $\alpha = 113°.5$, $\delta = +45°$. This is A.M.S. radiant No. 1956. The von Niessl-Hoffmeister Catalogue has no fireball from this vicinity near May 21.

Fireball of 1937 February 28/29

On this date at 7:40 P.M., E.S.T., a brilliant fireball was seen to the N.E. from New York City. We were fortunate in securing three reports from observers who know the constellations and gave coordinates, and two others which were confirmatory, qualitatively. In order we have: S1, I. L. Meyer, $\lambda = 74° 07'$, $\phi = 40° 50'$, $a_1 = 169°$, $\delta_1 = +45°$, $a_2 = 199°$, $\delta_2 = +46°$; S2, G. B. Kirkpatrick, $\lambda = 74° 00'$, $\phi = 40° 44'$, $a_1 = 180°$, $\delta_1 = +49°$, $a_x = 192°$, $\delta_x = +53°$, $a_2 = 201°$, $\delta_2 = +56°$; S4, C. A. Federer, $\lambda = 73° 15'$, $\phi = 42° 36'$, $a_y = 102°.6$, $\delta_y = -16°.6$, $a_x = 100°.6$, $\delta_x = -20°.6$; S3, —Miller, $\lambda = 74° 0' \pm$, $\phi = 41° 56' \pm$ "coming from S.E. . . . region of Orion"; S5, Miss Federer, $\lambda = 73° 57'$, $\phi = 40° 52' \pm$, $a_1 = 200° \pm$, $h_2 = 22½ \pm$.

The above coordinates were turned into azimuths and altitudes by Miss E. F. Reilly, who indeed made several approximate solutions which were of value to me in making this. At S2 we have a drawing, showing that the object exploded at X and the end was at a point given by $a_2 \delta_2$. At S4, Federer was in a car and did not see the first part of the path. The one he gives is very short. After some approximations, I projected his path backwards from his end point to 1.5 times the observed length. I took its upper end as his beginning point. As he was farther away, and speaks of no fragments, his "end" is taken as the explosion point of S2. I then secured three points on the path: beginning height from S1, S2, and S4 (corrected), explosion point from S2 and S4, and end point from S1 and S2. These three points lie almost on a straight line thus giving more confidence in the solution, despite the short projected path and the difficulties in properly combining the observations, which have just been described. As for the physical features, the color was blue; the magnitude at least —5 from S1 and S2; as stated it exploded near the end, but fragments continued. No long-enduring train was left; what there was rapidly disappeared. At explosion point the body is said to have had a diameter of about 15′. It is improbable that this fireball furnished meteorites, due to the height of the final disappearance of the small fragments. An examination of von Niessl-Hoffmeister Catalogue shows no fireball which could come from the radiant derived for this. The computed data follow.

Date	1937 Feb. 28/29, 7:40 P.M., E.S.T.
Sidereal time at end point	95°.2
Began over	$\lambda = 72° 59'$, $\phi = 41° 21'$ at 82.2 ± 8.6 km
Exploded over	$\lambda = 73° 06'$, $\phi = 41° 29'$ at 62.4 ± 2.4 km
Ended over	$\lambda = 73° 10'$, $\phi = 41° 33'$ at 47.6 ± 6.8 km
Length of path	44.5 km
Projected length of path	27.6 km
Observed velocity	18 km/sec (very uncertain)
Radiant (uncorrected)	$a = 327°$, $h = 51°.0$
Zenith correction (parab.)	—3°.8
Radiant (corrected)	$a = 327°$, $h = 47°.2$; $\alpha = 119°$, $\delta = +3°.6$

FIREBALL OF 1943 DECEMBER 18/19

On this date at 12:57½ A.M., E.S.T., a fireball was seen in the south from two stations. The condensed data follow. S1, Robert Angelini, $\lambda = 74° 00'$, $\phi = 40° 42'$, $a_1 = 4°$, $h_1 = 30°$, $a_2 = 16°$, $h_2 = 23°$ (these coordinates were measured from a diagram and transferred to a large celestial globe); streak 12° or 13° long, duration 10 to 15 sec., magnitude —8, bluish white. "The streak was faintly waved, as if by successive compression effects of the bolide on the atmosphere, the effects (5 or 6) becoming shorter and more rapid toward the end." S2, R. E. Franklin, $\lambda = 73° 01'$; $\phi = 40° 46'$ . . . brilliant white fireball, brightness > Venus and < Moon at last quarter, duration 5 seconds, leaving a 20° track of red sparks for 20 seconds. Trail from near κ Orionis through ½ $\mu\epsilon$ Leporis and on. From globe here we measure therefore $a_1 = 20°$, $h_1 = 38°$, $a_2 = 31°$, $h_2 = 19°$.

A graphical solution was then made, which however cannot be very accurate as the Δa's were only 16° and 15°, respectively. As is well known, larger differences in azimuth tend to give far better solutions. We find:

Date	1943 Dec. 18/19, 12:57½ E.S.T. = Dec. 19.25 G.C.T.
Sidereal time at end point	101°
Began over	$\lambda = 74° 13'$, $\phi = 38° 28'$ at 187 ± 37 km
Ended over	$\lambda = 75° 23'$, $\phi = 38° 00'$ at 140 ± 4 km
Length of path	115 km
Projected length of path	105 km
Observed velocity	23 km/sec (very uncertain)
Radiant (uncorrected)	$a = 241°$, $h = 24°$; $\alpha = 187°$, $\delta = +37°$
Zenith correction (parab.)	—0°.7
Radiant (corrected)	$a = 241°$, $h = 23°.3$, $\alpha = 188°$, $\delta = +36°$

Flower Observatory of the University of Pennsylvania, Upper Darby, Pennsylvania, 1945 February 14.

Contributions of the Society for Research on Meteorites

Edited by FREDERICK C. LEONARD
Department of Astronomy, University of California, Los Angeles 24

On the Cause of the Zodiacal Light and the Counterglow

FREDERICK C. LEONARD
Department of Astronomy, University of California, Los Angeles

It is sometimes stated in astronomical textbooks that the tiny meteorites that are responsible for the zodiacal light revolve around the Sun "nearly" in the plane of the ecliptic. The qualification is justified, if for no other reason than that the *median plane* of the zodiacal light is coincident doubtless with the *invariable plane* of the solar system—a fact that should without much difficulty be verifiable by observation. The only reason why the principal plane of the zodiacal light is "nearly" coincident with the plane of the Earth's orbit is undoubtedly that the latter plane is but slightly inclined to the invariable plane.[1]

The counterglow, on the other hand, is definitely related to the Earth, being due almost certainly to at least 2 things: (1) the greater brightness of the particles that produce the zodiacal light at a spot directly opposite the Sun, where, as viewed from the Earth, they present the full phase (A. Searle's explanation)

and (2) their greater concentration about a point some 930,000 miles from the center of the Earth, on the line joining the centers of the Sun and Earth, and beyond the vertex of its umbra, "where, in consequence of the combined forces of the Earth and Sun, wandering particles tend to circulate in a sort of dynamic whirlpool" (H. Gyldèn's and F. R. Moulton's explanation).[2] The second factor is a beautiful natural example of a Lagrangian straight-line solution of the problem of 3 bodies.

NOTES

[1] The invariable plane "is inclined 1° 35′ 8″ to the ecliptic of 1850 and has its ascending node in longitude 186° 9′," according to Russell, Dugan, & Stewart's "Astronomy," **1**, 285, 1926.

[2] *V.* Moulton's "An Introduction to Astronomy," New & Rev. Ed., 263-4, 1916, and also his "An Introduction to Celestial Mechanics," 2nd Rev. Ed., 305, 1914, and the 2 sources referred to therein.

On Tests for Nickel in Meteorites and Etching Metallic Meteorites

In the current January and February instalments of *C.S.R.M.*, a couple of tests for nickel in meteorites and specimens suspected of being such were given. To these may be added the following tests, quoted from O. C. Farrington's "Meteorites," 3, 1915:

"A convenient test for nickel in a mass whose meteoritic origin is suspected may be made by dissolving a fragment of the substance to be tested in nitric acid, adding ammonia till the acid is neutralized, boiling, adding a little more ammonia to make sure that all the iron has been precipitated, and filtering off this precipitate (ferric hydroxide). If the filtrate shows a bluish tinge, the presence of nickel is indicated, but, as small amounts of nickel might not be indicated in this way, a few drops of yellow ammonium sulfide should be added to the cold, clear filtrate. If nickel is present, a black precipitate of nickel sulfide will be obtained. In order to test this further, the liquid should be filtered and the precipitate tested with a borax bead. If nickel is present, a violet bead will be obtained in the oxidizing flame, changing to reddish-brown on cooling. Another test for nickel in the presence of iron consists in dissolving the substance to be investigated in hydrochloric acid, boiling for a moment with a few drops of nitric acid to oxidize the iron, adding a little citric or tartaric acid to prevent precipitation of the iron, neutralizing the solution with ammonia, and adding a few drops of a solution of dimethylglyoxime in alcohol. If nickel is present, a blood-red color will be given the solution; if not, no change of color will occur."

Instructions in the art of etching metallic meteorites (*cf.* Dr. H. H. Nininger's paper on this subject in the February instalment), based mostly on the method of the Foote Mineral Company, are contained on pp. 127-9 of Farrington's book.

Retirement of Henry W. Nichols

Henry W. Nichols, Chief Curator of the Department of Geology of the Chicago Natural History Museum, retired, on account of ill health, on December 31, 1944, after serving for 50 years. Mr. Nichols, who is in his 78th year, joined the staff in 1894, as Curator of Economic Geology. From 1897 to 1921, he was Assistant Curator of Geology; from 1921 to 1933, Associate Curator; from 1933 to 1936, Curator of Geology; and from 1936, Chief Curator. He has conducted 16 expeditions in North and South America for the Museum, collecting a wide

variety of geological material. Dr. Paul O. McGrew has been appointed Acting Chief Curator of the Department.—*Science, N. S.*, **101**, 35, January 12, 1945.

Mr. Nichols is a charter member and an original fellow of the Society for Research on Meteorites. He served as a Councilor of the Society during the first two terms of its existence, *i.e.*, from 1933 until 1941. Our best wishes go with him as he retires from active duty, after a long and useful career.

President of the Society: LINCOLN LA PAZ, Department of Mathematics, Ohio State University, Columbus 10

Secretary of the Society: C. H. CLEMINSHAW, Griffith Observatory, P. O. Box 9866, Los Feliz Station, Los Angeles 27, California

Comet Notes

By G. VAN BIESBROECK

The only comet under observation at this time is PERIODIC COMET SCHWASSMANN-WACHMANN 1925 II. As far as the incomplete records of this winter indicate, this object has remained faint most of the time. When last recorded here (February 1, 1945) it appeared as a small nebulosity not brighter than a star of magnitude 17. The present observing season will soon come to a close as the comet moves into the evening sky.

Before long the well-known PERIODIC COMET PONS-WINNECKE is due to return to visibility. Its history goes back to 1819 when it was found by Pons at Marseilles (France). While the periodic nature of the orbit was recognized at that time, the comet was missed at subsequent oppositions until it was rediscovered by Winnecke at Bonn (Germany) in 1858. In view of this year's return the computing section of the British Astronomical section has derived a new orbit from the observations at the most recent appearance in 1939. Planetary perturbations since that date were taken into account in establishing the following ephemeris based on the assumption of 1945 July 8.02 U.T. as the time of perihelion (*Handbook of the B. A. A.*, 1945, p. 34).

1945	α h m	δ ° ′	Distance from Sun	Distance from Earth
Mar. 2	13 45.9	+32 2	1.906	1.092
10	49.2	34 42		
18	50.3	37 25	1.776	0.932
26	49.0	40 3		
Apr. 3	45.1	42 23	1.649	0.812
11	38.9	44 15		
19	30.8	45 27	1.526	0.721
27	22.1	45 48		
May 5	13 13.8	+45 13	1.411	0.647

The comet is favorably located in the constellation of the Hunting Dogs, but it will be quite faint during this early period. The comet will be brightest in July when the minimum distance from the earth is reduced to 0.41 astronomical unit, but it is not expected to be brighter than magnitude 9 which it reached in 1939 when it came somewhat closer to the earth.

Two other periodic comets will be looked for in March. However, they are both faint and none too favorably located. They are COMETS VÄISÄLÄ 1944 *b* and OTERMA 1943 *a*, both discovered recently at the Tartu (Finland) Observatory.

Williams Bay, Wisconsin, 1945 February 14.

Communications and Comments

Under this heading we shall publish from time to time such material as does not properly fall under any of the established headings of this journal. Here, too, may be found, when occasion arises, articles which the editor may not be willing to give sanction to but which, nevertheless, may be provocative of thought along new lines.

An Extra-Galactic Suggestion

The suggested curvature of space should predicate the like for line of sight, so that, looking out from our Universe in one direction, we should see its back, namely as seen from the opposite point in the sky. In truly spherical curvature, these reflections should be seen in all directions, giving the whole sky a luminous background, interrupted only by obscuring matter or other galactic objects, and dotted by the external spiral nebulae. Since this is not apparent, any curvature might be ellipsoidal, for which the lines of reflection would be confined to one of the equatorial planes, or parallels, of the ellipsoid. In rough analogy one dimension down, while on "straight" lines (great circles) from any point on a sphere return thither, such lines (other than the equatorials and parallels) on an ellipsoid would spiral away on their "straight" course, ever approaching one of the axial poles, away from their origin.

If not seeking a needle in a haystack, this might be tested for antipodally paired right- and left-handed images indicative of our own Galaxy, by photographs such as those from the 100-inch telescope at Mt. Wilson, or in those to be taken with the 200-inch on Mt. Palomar. Any such reflections might be recognized by the relative positions of our Magellanic Clouds, according to direction of sight.

So alone could we "see ourselves as others see us"—tilted according to the axes of space curvature, which latter could be derived from these reflections. The scale of curvature would follow from angular diameters (if measurable) or brightness of such reflections.

This, of course, takes no account of the Time element: whether our Galaxy be old enough for its image to have made the supposed circuit. Also, in this connection, the change in its appearance meanwhile, should the theory of galactic development be true.

The foregoing develops somewhat Sir J. Jeans' qualified remark ("The Stars in their Courses," 1931, p. 130) about antipodal reflections of M. 31 and M. 33. If these be real, the whole circuital distance could follow from comparison between the real and the reflected images in each case.

So generally, perhaps at last, we reach the inner meaning of the phrase (Prov. 8.27) "In His decreeing a circuit on the face of the deep" (so Heb.) revealed three thousand years ago in the Book, so often limited, not in itself, but in our own understanding.

O. R. Walkey.
Stanford Bishop Vicarage, Bringsty, Worcester, England, March, 1944.

General Notes

Readers familiar with this magazine may receive the impression that this number is smaller than is usually the case. This impression is partly wrong and partly right. We are using a lighter grade of paper on the inside and for the cover, and also have a smaller number of pages, both features contributing to the impression, and both the result of war restrictions on materials and indirectly upon the astronomical output. This, like many other present situations, we hope, is only temporary. EDITOR.

The Origin of Satellites

A great number of the bodies which make up the universe, or that part of it which is known to us, is in a state of orderly revolution. Where two bodies of fairly equivalent mass revolve round one another, such as the binary stars, the explanation is simple and satisfactory, namely, fission in a liquid state.

When we come to consider the case of a relatively small body revolving round one of much greater mass, there are two possible ways in which the system may have originated. Firstly, the satellite may have been a wandering body, captured by the parent on approaching so near that its own velocity was insufficient to overcome the gravitational attraction of the latter. It has been argued that this is the history of all satellites. If so, we should expect to find something like an equal number revolving in each direction.

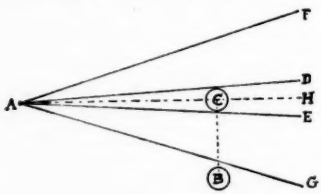

FIGURE 1

Consider a relatively small body approaching a large one from the point A in Figure 1; the larger being momentarily at B. Suppose that, by the time the small one has covered the distance between them, the larger one has arrived at C. If the path from A lies between AD and AE, there will of course be a collision. If it lies outside some angle FAG, the small one will evade capture and pass away, having merely undergone a large or small deflection from its original course. If it approaches on a line lying between AF and AD, it will be caught, and will begin to revolve round C in a clockwise direction; if between AE and AG, the same thing will happen, except that it will revolve counter-clockwise. The magnitudes of the limiting angles FAH and HAG depend on the masses and speeds in any given case, but it is obvious that they must be equal to one another, or very nearly so, whence it follows that satellites caught in this way are just as likely to go round one way as the other. But we find that in fact, out of 27 known satellites, only four are retrograde. In any particular case, therefore, the odds are much against this theory. Strictly speaking, Figure 1 is not quite

accurate, because all the possible paths would be more or less curved, owing to the attraction of the larger body; moreover, this attraction in the region FAD would differ slightly from that in EAG, but these considerations are insufficient to affect the argument materially.

The second possibility, and the one which, it is submitted, covers by far the greater number of cases, is that the satellite has been thrown off from the parent at some past time, like a spark from a grindstone, at a speed insufficient to send it altogether clear. Its orbital motion would be derived from the rotation of the parent. Its own rotation can be accounted for by the fact that, at the moment of separation, its outer part would have been moving more rapidly than the inner part, a state of affairs which would set up a spin.

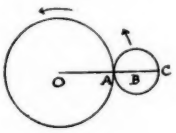

FIGURE 2

In Figure 2, let the circles represent, say, the earth and the moon. Let V be the speed at A, relative to the point O. Then, if r_e and r_m are the radii of the earth and moon, the speed at B is $[(r_e + r_m)/r_e]$ V, and that at C is $[(r_e + 2r_m)/r_e]$ V. Therefore the speed at C, relative to B, is (r_m/r_e) V. If the ratio $r_m : r_e$ was the same as at present, it would have been (1079.8/3963.339) V, = .272 V. The moon may not, of course, have been spherical at first, in which case the calculation fails, but the principle remains. It is interesting to note that the point A on the moon moves upwards with reference to O, and downward relative to B.

A wandering body required by the first theory would have obtained its rotational and progressive motions just in the same way, as it would presumably have been originally thrown off by some other body, sufficiently fast to escape the sphere of influence of the latter.

Owing to the effects of tidal action, the rotational speeds of bodies in the solar system tend to slow down. In the case of a satelltie and its parent, they do so at very unequal rates. At the present time, a point on the moon's equator only travels about 10¼ miles an hour, relative to the centre. The effect of contraction is that, although the angular velocity is increased, the linear or tangential speed remains the same.

When a satellite is thrown off, there are four things which might conceivably happen. It might (1) fall back on the parent, (2) revolve round it at a constant radius, (3) revolve continually in an elliptic orbit, or (4) escape altogether. There are two critical speeds; one at which it would (theoretically) take the second course, and the other at which it would just escape. As a matter of fact, it never would take the second course, because it is impossible to imagine that it would go off at the precise speed required, and maintain that speed; nevertheless, this speed marks the boundary between (1) and (3). In order to find it, we can put the centrifugal force equal to the weight, thus: $Wv^2/gR = W$, whence $v = V(gR)$; R being the radius of the path in feet.

If it is going just fast enough to escape, say u feet per second, it will travel in a parabolic path, the focus of which is the centre of the parent. From the nature of the parabola, the radius of curvature at the vertex (which will be the starting

point) is twice the focal distance and equals 2R. We have therefore $Wu^2/2gR = W$, and $u = \sqrt{(2gR)}$.

As R is increased, these speeds become less. Let R be augmented in the ratio 1 :n, where n is greater than unity. By the law of inverse squares, g is divided by n^2 and gR by n, so that the speeds are divided by $\sqrt{(n)}$.

Suppose the earth and moon are smooth spheres, and the latter starts just clear of the former; there being no atmosphere to offer resistance.

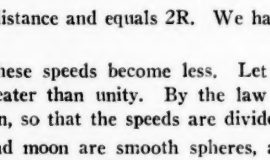

FIGURE 3

Let the centre of the moon start from A in Figure 3, with a speed v feet per second; B being the point at which it would arrive at the end of one second in the absence of gravity, so that AB = v. With gravity taken into account, if it is going at the lower of the two critical speeds, namely $\sqrt{(gR)}$, it will go to C, so that OA and OC are radii of the circular orbit, and are equal to the sum of the equatorial radii at the time. BC, the distance the moon is pulled in by gravity, is numerically equal to ½g. Here $AB^2 = OB^2 - OA^2 = ½g\ (2R + ½g)$, $= gR$ we can say, since ½g is so small compared with 2R, and $v = gR$, as before.

Let AD = u; AE being the smallest parabolic path and DE = BC = ½g, as before. By a similar process of reasoning, $AD^2 = ½g\ (4R + ½g)$ = say 2gR, giving $u = \sqrt{(2gR)}$. Taking R at its present value (3963.339 + 1079.8 = 5043.139 miles) = 26,627,780 feet, g would be reduced to 19.82, being 32.091 (the value at the equator) multiplied by $3963.339^2 \div 5043.139^2$. The critical speeds in these circumstances would be 15,663 and 22,151 miles an hour. It is, however, probable that R was very much greater at the time of the rupture, in which case the speeds would be correspondingly reduced.

If we knew the average rates of change of speed with great accuracy, it would be a simple matter to say when the catastrophe occurred. For we could write $A + bx = C + dx$, whence $x = (A - C)/(d - b)$. Here A is the present orbital speed of the satellite and b, its average annual deceleration; x being the number of years that have elapsed since it broke away. C is the present linear rotational speed of a point on the parental equator, and d, its average yearly retardation, because, by the hypothesis, A and C would originally have been equal to one another.

Again taking the case of the earth and moon; the most recent estimate for A is 2,287 miles an hour. The circumference of the earth at the equator is 24,902 miles. Dividing by 24 gives 1037 miles an hour, so that A — C = 1250. Two things are obvious, one being that $d > b$ and the other that d — b is a very minute quantity. Yet, considering the great differences in mass, velocity, orbit and probable rate of cooling of the earth and moon, the difference between d and b must be very substantial, compared to their magnitudes. Hence they must both be very small. If we put 2,000 millions of years for x, which is by no means an

outside estimate,* d — b = 0.04 of an inch, or 1 millimeter, per hour, per year.

Treating the sun as the parent and the earth as the satellite; the mean orbital speed of the latter is 18.5 miles a second, therefore 66,600 miles an hour. The sun's diameter is approximately 865,000 miles; circumference, 2,171,480 miles. It rotates in 26 days, hence a point on the equator travels (relatively to the centre) 104,514 miles a day, or 4,355 miles an hour. In this case A — C = 62,245. Taking x again as 2,000 million years (on the supposition that the moon was thrown off only very shortly after the earth had separated from the sun, being pulled out by the sun's attraction at a moment when the earth approached it exceptionally closely, before setting down to an orbit at a safe distance), we find that d — b is about 20 inches per hour per year.

To glance briefly at the various systems in order; proceeding outwards from the sun, we have Mercury and Venus with no known satellites; we then come to the earth with one, a peculiarity of which is its relatively large mass, the ratio of which to that of the earth is 1 to 81.53; whereas the satellite with the next largest proportion of mass to that of its parent is in the system of Saturn, where Titan has 1/4000 of the parental mass. This observation merely applies to mass relative to that of the parent; as to actual mass, most of Jupiter's moons are larger than ours. This very marked discrepancy must surely be due to some profound difference in the history of our moon from that of the rest; it may have originated at a much later period of the evolution of its parent than was the case with the others.

The first planet outside the earth's orbit is Mars, with two satellites, Deimos and Phobos. The latter, which is the inner one of the two, revolves in less than a third of the diurnal rotation of Mars, so was probably a capture; very likely an asteroid; Deimos, in six days longer than the Martian day.

Next come the asteroids, which are thought, not without good reasons, to be the fragments of a large primeval planet which, for some reason, burst, shrapnel-fashion.

Proceeding on our way, we come to Jupiter, with no less than eleven satellites, two of which revolve in a retrograde direction, and can, at all events, be handed over to the first theory. Jupiter is especially well-equipped for capturing outsiders, on account of its huge mass, which exceeds that of the whole of the rest of the planets in the proportion 5 to 2, and gives him enormous attractive power. It is also favorably situated, with an orbit next door to the asteroids. The two retrograde moons were probably gathered in from them.

Saturn possesses nine satellites, one of which, Phoebe, is retrograde, and therefore probably an *enfant trouvée*.

Uranus has four, which have the remarkable peculiarity of revolving in a plane nearly perpendicular to that of the orbit of their parent.

Neptune is known to have one, which goes round in a retrograde direction, and was, therefore, probably caught up from somewhere.

Pluto is not known to have one.

It seems fairly certain that the retrograde satellites have been gathered in from outside. As there are four of them, the chances are in favor of there being about the same number among those which are going round the normal

*Jeans, 2,000 millions; Jeffreys, double that amount.

way, of which Mars appears to possess one; the rest are probably members of the families of Jupiter or Saturn, owing to their large masses and consequently powerful gravitational fields.

If we accept the hypotheses that the moon was thrown off the earth and the earth off the sun, may not the sun have originated in the same way? It may be travelling around some inconceivably remote primeval centre.

We are even taught that the universe is expanding. If this process has been continuous for a long enough time, it must have originated from a point—perhaps one should not say a point in space, for space itself may not have existed then!

C. F. Dendy Marshall.

Book Reviews

Direction Finding by the Stars, by J. B. Sidgwick. (Faber and Faber, 24 Russell Square, London.)

This is a pocket-sized book of 88 pages which sets forth the facts related to the changing aspects of the sky from hour to hour and from season to season. The person who is conversant with these facts is able to make use of them to his advantage should he find himself in strange surroundings where landmarks are no longer significant to him in the determination of directions. Under such circumstances, one needs only a general idea concerning directions, which is the purpose of the book. Refinements which can be introduced by means of instruments are omitted.

Obviously, such knowledge will be of the same or greater value to one who is adrift at sea. Since many are now living under conditions which might lead to such a situation the lore of this book has particular significance for this generation. The author implies that this circumstance was one factor which impelled him in the preparation of it.

It is clear, concise, and intelligible to any one who devotes a little time and care to a study of it. C.H.G.

Five-Figure Logarithm Tables. (His Majesty's Stationery Office, London, England. 7s 6d.)

A collection of three sets of five-place logarithms has recently been made available as indicated above. The collection consists of reproductions of tables issued earlier in other connections. They are the following:

1. Chappell's table of logarithms, giving 5-figure logarithms of the numbers 10000 to 40000 and 4000 to 10000.

2. von Rohr's table of 5-figure logarithms of sines and tangents of the angles 0°.000 to 5°.000, for each one-thousandth of a degree.

3. Bremiker's table of 5-figure logarithms of sines, tangents, cotangents, and cosines of the angles 0°.00 to 45°.00, for each one-hundredth of a degree.

Persons familiar with mathematical tables will recognize these to be among the best. The compilation here affords the use of these in a convenient form. They are bound in dark-blue, durable fabricoid, and will surely be found serviceable by computers who have not as yet delegated the arduous details of computing to the seemingly intelligent computing machines. C.H.G.

Handbook for 1945, The Observers, Published by the New Zealand Astronomical Society.

A copy of the first issue of this Handbook from this source has recently been received. It consists of 32 pages of descriptive material and illustrations. There are several photographs of the heavens and a number of charts to aid in recognizing the stars. A conspicuous configuration in these photographs is that of the Southern Cross. This is as it should be for observers in New Zealand, but it seems strange and unusual to those who spend their lives north of the equator. The material it contains will be found very useful by amateurs. The New Zealand Astronomical Society is to be commended for preparing this convenient pamphlet for use by its members and constituents. C.H.G.

Lover of Peaceful, Starry Lanes

Compound of matter and ebullient mind,
Of blood and breath and soul,
Flesh and spirit, in man combined
Into one enigmatic whole.

His mire-bound feet cannot ascend
Where star-trails intermesh;
The finer mind can't comprehend
The coarser ways of flesh.

Enchained to earth, distraught in mind,
Aghast at the world upheavals,
He racks his brain yet fails to find
A cure for the disastrous evils.

Lover of peaceful, starry lanes
Yet scion of a war-mad clan—
Straining against atavistic chains—
Tradition-bound, truth-hungry MAN.

—Imo Lucile Reece.

Amsterdam, N. Y.

POPULAR ASTRONOMY

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